Electrical resistance of quantum nanowires with disorder

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Overview

Motivation : transport experiments in nanowires

Our model

Results

Conclusion

Appendices

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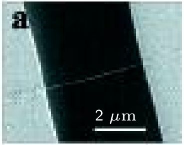
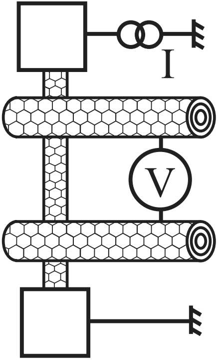
Framework and aim

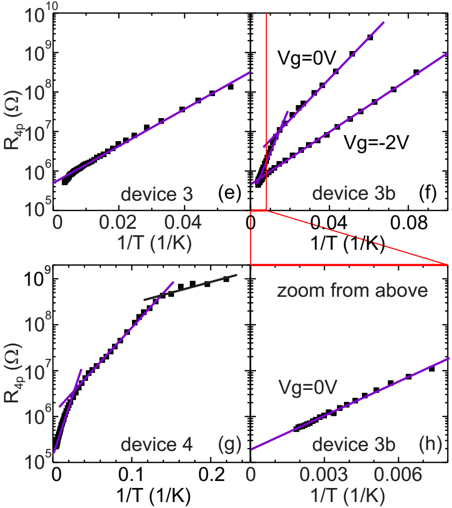


Y. Huang *et al.*

Aim : understand charge transport in some 1D materials : nanowires, nanotubes, DNA...
Cheap, mass production of nanodevices $\rightsquigarrow$ deal with defects

Experiment : Carbon NanoTubes





B. Gao *et al.* Phys. Rev. B 2006

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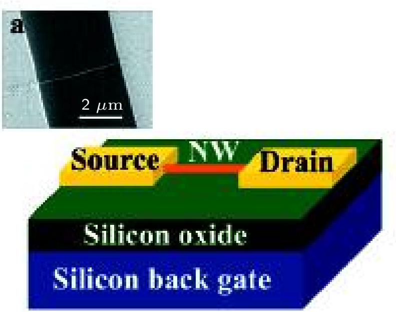
Interactions assumed short range

- We ignore spin. We are in steady regime (DC).

- Electrostatic interactions are long range, but we assume screening :



Y. Huang *et al.*

Capacity per unit length : C

Tomonaga-Luttinger liquids

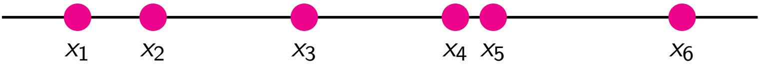
Special effect in 1D
Parameter $K \leftrightarrow$ intensity of interactions between electrons :

$K < 1$	repulsive interactions
$K = 1$	no interactions
$K > 1$	attractive interactions

For us, $K < 1$. For C nanotubes, $K \approx 0.3$; K may be ≈ 0.15 for nanowires.

M. Shiraishi *et al.* Solid State Comm. (2003)
L. Venkataraman *et al.* PRL (2006)

Model of the disorder

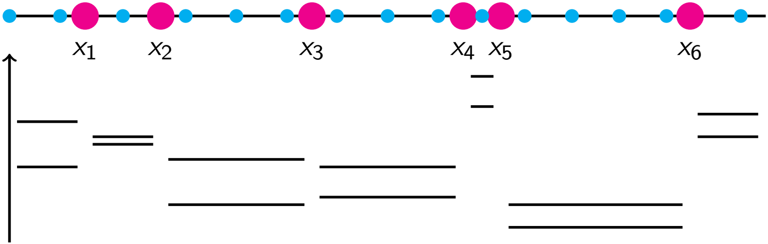


Strong, point-like impurities distributed randomly according to a Poisson process with cut-off. Between : pure Luttinger liquids.

Chain of quantum dots

- Without impurities : uniform density n of electrons.

- With impurities + gate electric field :

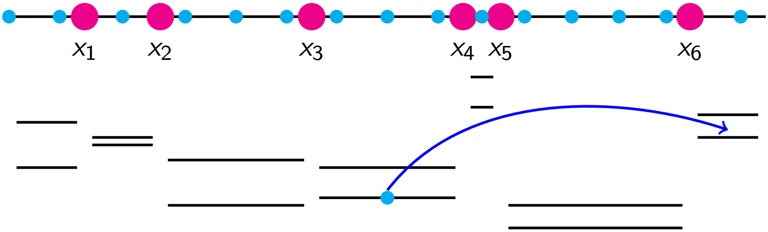


energy levels with respect to ground state of dot = costs to add or remove an electron

Nattermann *et al.* 2003, Fogler *et al.* 2004, Malinin *et al.* 2004

Chain of quantum dots : conduction mechanism

When an electron leaves a dot and reaches another one :



- It has to borrow to the phonons the energy difference of states $\rightarrow P \sim e^{-\Delta E/T}$;
- It has to cross impurities by tunneling. Computation for a Luttinger liquid $\rightarrow$ transparency e^{-s} for one impurity, thus $(e^{-s})^n$ for cotunneling through n impurities. Larkin & Lee 1978
- Decoherence after each hop (dissipation).

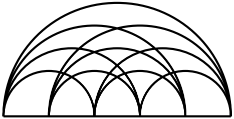
Competition between tunneling and activation

Equivalent resistor circuit in the low field regime

A. Miller & E. Abrahams 1960

If the applied voltage is small, current I is prop. to voltage V and one may define resistances :

$$R_{ij} := \frac{\xi_i - \xi_j}{I_{i \to j}} \approx R_0 e^{\frac{E_{ij}}{T} + s|j-i|} \; .$$



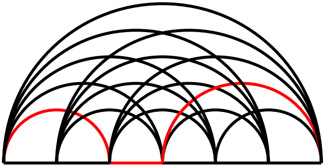
$\rightarrow$ Resistor network (with T dependance) with disorder

We performed simulations to compute the total R (not completely straightforward but works quick and fine up to more than 10000 impurities).

"Percolating path" approximation

At low temperatures, it is often claimed that R is given by R of the "percolating path".

Ambegaokar *et al.* Phys. Rev. B (1971)



- Our numerics → this approximation is very good up to $T \approx 100\Delta$. Δ : typical charging energy of one dot.
- Therefore we will discuss properties of the percolating path because it's easier.

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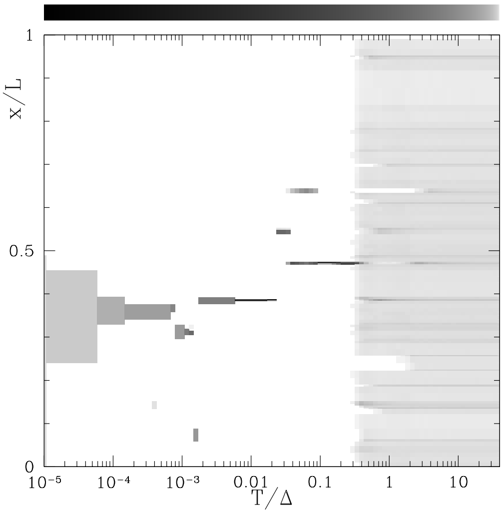
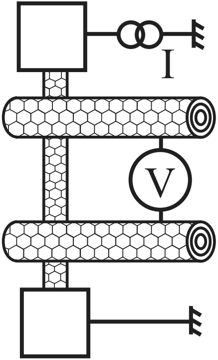
Numerical insight for one sample : potential gradient

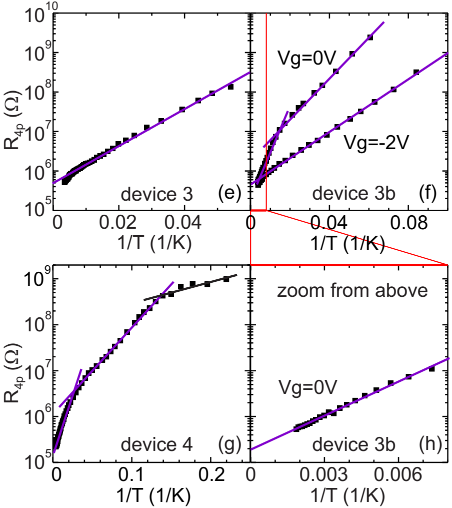
$K = 0.1$
$N = 100$ impurities

2 regimes :
extensive / non
extensive R



Experiment : Carbon NanoTubes

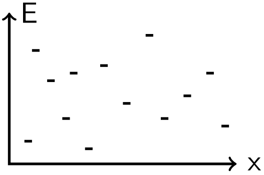




B. Gao *et al.* Phys. Rev. B 2006

Mott's Variable Range Hopping (1)

Model for disordered semiconductors :

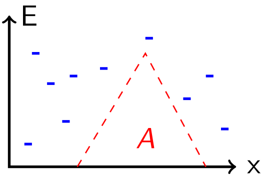


Localized states (LS) uniformly random in space (x) and energy (E)
Hopping conductivity

Mott's Variable Range Hopping (2) : 1D breaks

In 1D, resistance of the sample is not fixed by the *typical* resistance but by *rare events*.

Kurkijärvi PRB 1973 ; Raikh & Ruzin JETP 1989



Break of resistance $R_0 e^u$ has probability $\propto e^{-Tu^2}$ to occur in each position.
Strongest break that occurs in the wire of length L :

$$Le^{-Tu^2} \approx 1 \rightarrow u \approx \sqrt{\ln(L)/T}$$

thus total resistance is $\approx R_0 e^{\sqrt{\ln(L)}/\sqrt{T}}$.

Breaks in our model (1)

Difference with Raikh & Ruzin : correlation between x and E [and strong interactions between electrons].

Probability of a break of resistance $R_0 e^u$:

$$\ln P(u) \approx \begin{cases} -\frac{1}{s}\frac{T}{\Delta}u^2 & \text{when } u \ll \frac{\Delta}{T} \\ -\frac{1}{s}u\ln\frac{uT}{\Delta} & \text{when } u \gg \frac{\Delta}{T}. \end{cases}$$

Strongest break that may be observed in a wire of length L given by $u_{\max}$ such that :

$$L\int_{u_{\max}}^{+\infty} P(u)\mathrm{d}u \approx 1.$$

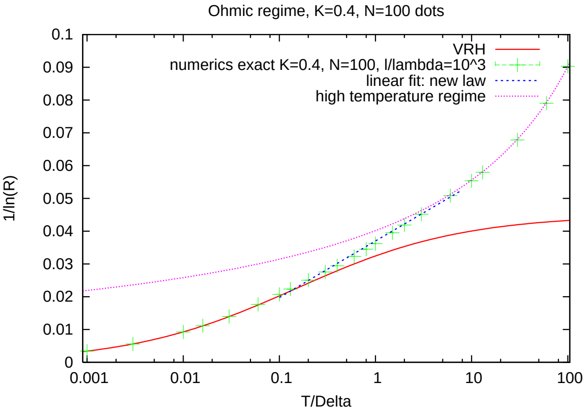
Breaks in our model (2)

Hence

$$\text{average } \ln R \sim \begin{cases} \sqrt{\frac{\Delta}{T}} & \text{when } \frac{T}{\Delta} \ll \frac{1}{s}\frac{1}{\ln L} \\ s\frac{\ln L}{\ln \frac{T}{\Delta}} & \text{when } \frac{1}{s}\frac{1}{\ln L} \ll \frac{T}{\Delta} \ll \frac{1}{s} \end{cases}$$

Fogler, Malinin & Nattermann Phys. Rev. Lett. 2006

Breaks in our model (3)

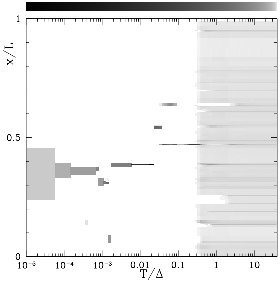


Numerically confirmed, *but*

- The two regimes may be hard to distinguish
- Huge sample to sample fluctuations

Distribution of ln R (1)

$$R_{\rm wire} \approx R_{\rm perc.path} = R_1 + R_2 + \ldots + R_k$$



$R_i \approx e^{n_i s + \frac{\Delta_i}{T}}$ is random $\Rightarrow$

Gaussian $R_{\rm wire}$ if T high or L (thus k) large

$R_{\rm wire} \approx \max(R_i)$ if T small and L not too large. Or : $\ln R_{\rm wire} \approx \max(\ln R_i)$.

Extreme value statistics :

Fisher-Tippett-Fréchet-Weibull-Gumbel 1920's ; Gnedenko 1941

$\max(\ln R_1, \ln R_2, \ldots, \ln R_k)$ is not Gaussian ; follows (in our case) Gumbel law

Distribution of $\ln R$ (2)

Finite-size effects : P. Hall 1978 ; . . . ; Z. Rácz & coworkers 2000's

- at low T, $\ln P(u) \propto -\frac{1}{s}\frac{T}{\Delta}u^2$ (well verified in the simulation) $\Rightarrow$ $\ln R \sim \sqrt{\ln L} \pm 1/\sqrt{\ln L}$

known in standard VRH : Lee *et al.* 1984, 1986

$$\boxed{P(\ln R) = \mathrm{Gumbel} + \text{f.-s. corrections in } 1/\ln L}$$ new

The corrections disagree with results from Raikh & Ruzin (1989)

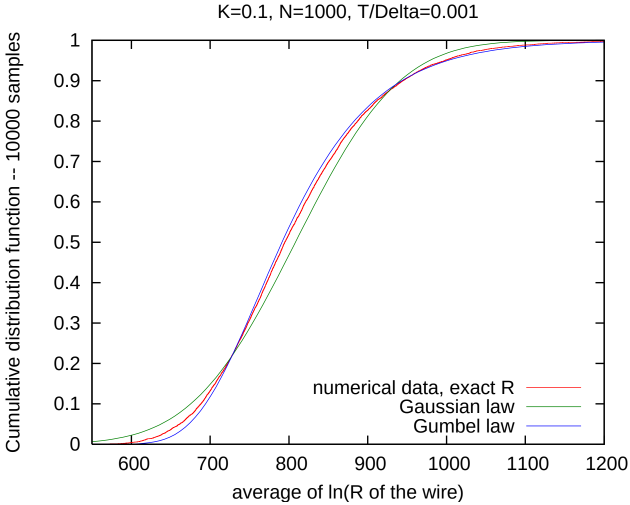
- at medium T, and if interactions (K) are not too large, $\ln P(u) \propto -\frac{1}{s} u \ln \frac{uT}{\Delta} \Rightarrow \ln R \sim \ln L/\ln\ln L \pm \ln\ln L/\ln L$,

$$\boxed{P(\ln R) = \mathrm{Gumbel} + \text{f.-s. corrections in } 1/\ln L/\ln(s\tfrac{T}{\Delta}\ln L)}$$

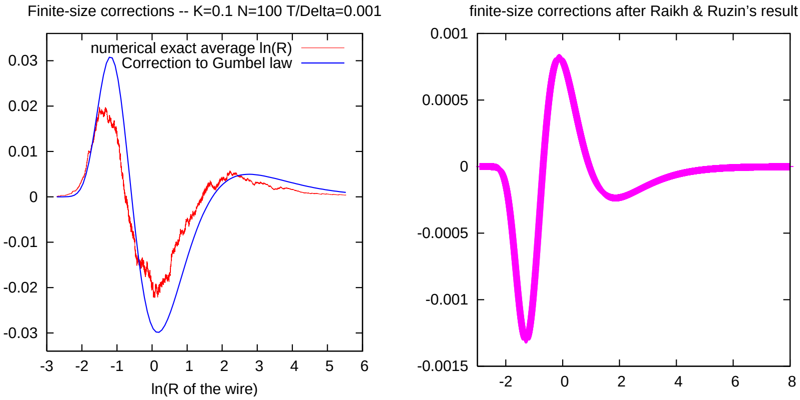
(new).

Suggestion to experimentalists : finite-size corrections to distributions can be a proof of (Luttinger liquid + strong impurities)

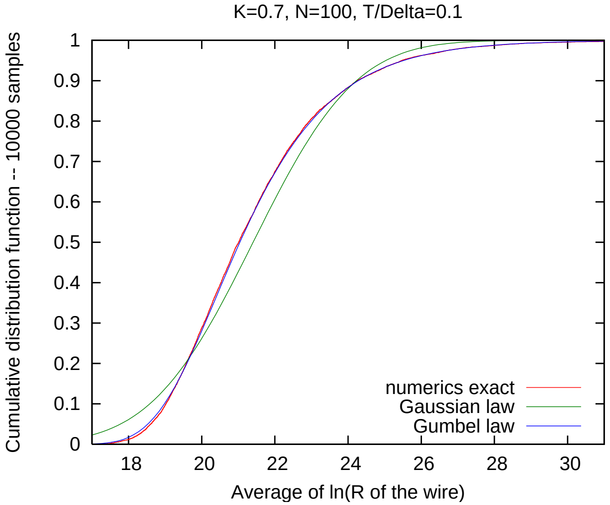
Distribution of ln R : numerics low T



Distribution of $\ln R$: numerics low T — f.-s. corrections



Distribution of ln R : numerics medium T



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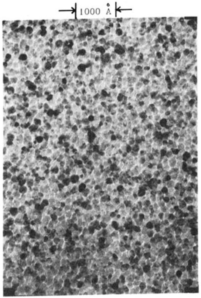
- Rich behaviour of (quasi) 1D quantum wires with disorder : Several temperature regimes, non trivial statistics
- Looking at the distribution of R may confirm/infirm the theory $\rightarrow$ call for systematic measurements

Perspectives

- Going to the non ohmic regime (strong electric field, non linear)
- Several coupled chains
- Could these results extend to granular metal arrays ?
- Studying the distribution of current and shot noise



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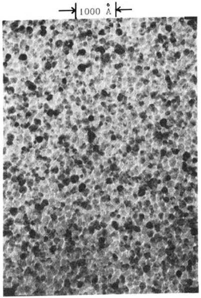

Thanks to T. Nattermann and M. Fogler !

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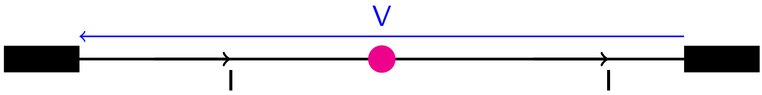
Effect of a single impurity / many weak impurities



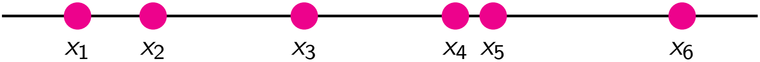
$$I \propto \begin{cases} V^{b+1} & V \gg 1 \\ T^a V & V \ll 1 \end{cases} \qquad \text{with} \qquad a = b = 2\left(\frac{1}{K} - 1\right).$$

Kane & Fisher 1992

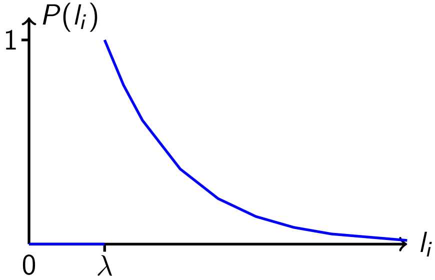
Same behaviour for many weak impurities.

Experimentally verified on short wires ($L < 1\mu\mathrm{m}$) or on relatively clean wires. But on some long wires ($L > 10\mu\mathrm{m}$), a/b is between 2 and 5 : effect of strong disorder ?

Model of the disorder



Strong, point-like impurities distributed randomly according to a Poisson process with cut-off. Between : pure Luttinger liquids. Probability of the distance $l_i = x_{i+1} - x_i$ between impurities i and $i+1$:



$$\langle l_i \rangle = l \gg \lambda$$

$$\forall i \quad l_i > \lambda$$

Hopping current between two dots (1)

A. Miller & E. Abrahams 1960

Net flow from state i to state j :

$$I_{i\to j} = f_i(1-f_j)w_{ij} - f_j(1-f_i)w_{ji}$$

where f_i is the occupation probability of state i by an electron (fermion) :

$$f_i = \left[\exp\frac{E_i-\mu_i}{k_\mathrm{B}T}+1\right]^{-1}$$

and w_{ij} is the "attempt frequency" to go from i to j :

$$w_{ij} = \mathrm{sign}(E_j-E_i)|\gamma| \underbrace{e^{-s|i-j|}}_{\text{tunnel effect}} \underbrace{\left[e^{\frac{E_j-E_i}{k_\mathrm{B}T}}-1\right]^{-1}}_{\text{borrowing from phonons}}$$

Hopping current between two dots (2)

At low temperatures,

$$I_{i\to j} \sim I_0 e^{-s|j-i|} e^{-\frac{E_{ij}}{2T}} \sinh \frac{\xi_i - \xi_k}{2T}$$

with

$$E_{ij} := \min_{\sigma,\tau=\pm} \left(|E_i^\sigma - \mu_i| + |E_j^\tau - \mu_j| + |E_j^\tau - E_i^\sigma| \right)$$

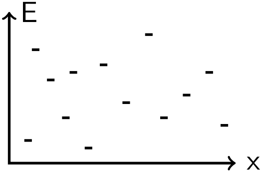
and, in the Ohmic (linear) regime where I is proportional to the applied field F or to the difference of electrochemical potential, we define

$$R_{ij} := \frac{\xi_i - \xi_j}{I_{i\to j}} \approx R_0 e^{\frac{E_{ij}}{T} + s|j-i|} \ .$$

$\rightarrow$ We now have a disordered network of resistances which depend on T.

Mott's Variable Range Hopping (1)

Model for disordered semiconductors :



Localized states (LS) uniformly random in space (x) and energy (E)
Hopping conductivity
Localization length a

Optimize *typical* conductivity $G = G_0 e^{-\frac{|x_j - x_i|}{a} - \frac{\Delta E}{T}}$ (with $\Delta E = E_j - E_i > 0$) :
in the ball of radius r with density of states ρ, typically ρr^D states $\rightarrow$ lowest $\Delta E \sim 1/(\rho r^D)$ in dimension D.

Optimizing over $r \rightarrow G \sim e^{-\left(\frac{T_0}{T}\right)^{\frac{1}{1+D}}}$.

In 3D, well known experimental result $R \sim e^{\left(\frac{T_0}{T}\right)^{\frac{1}{4}}}$.

Breaks in our model (2)

Hence

$$\ln R \approx \begin{cases} \sqrt{s\frac{\Delta}{T}\ln\frac{L}{l_0}} & \text{when } T \ll \frac{\Delta}{s}\frac{1}{\ln\frac{L}{l_0}} \\ s\frac{\ln\frac{L}{l_1}}{\ln sY\frac{T}{\Delta}} & \text{when } \frac{\Delta}{s}\frac{1}{\ln\frac{L}{l_0}} \ll T \ll \frac{\Delta}{s}\frac{L}{\lambda} \end{cases}$$

X, Y, l_0, l_1 : constants or slowly varying functions of T.

“Phase diagram” : (non) extensitivy of R