Electrical resistance of quantum nanowires with disorder

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Overview

Motivation: transport experiments in nanowires

Our model

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Appendices
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Framework and aim

Aim : understand charge transport in some 1D materials: nanowires, nanotubes, DNA...
Cheap, mass production of nanodevices $\Rightarrow$ deal with defects
Experiment: Carbon NanoTubes
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Interactions assumed short range

- We ignore spin. We are in steady regime (DC).

- Electrostatic interactions are long range, but we assume screening:

  Capacity per unit length : $C$
Tomonaga-Luttinger liquids

Special effect in 1D
Parameter $K \leftrightarrow$ intensity of interactions between electrons:

- $K < 1$ repulsive interactions
- $K = 1$ no interactions
- $K > 1$ attractive interactions

For us, $K < 1$. For C nanotubes, $K \approx 0.3$; $K$ may be $\approx 0.15$ for nanowires.

L. Venkataraman et al. PRL (2006)
Model of the disorder

Strong, point-like impurities distributed randomly according to a Poisson process with cut-off. Between: pure Luttinger liquids.
Chain of quantum dots

- Without impurities: uniform density $n$ of electrons.

- With impurities + gate electric field:

  energy levels with respect to ground state of dot = costs to add or remove an electron

Nattermann et al. 2003, Fogler et al. 2004, Malinin et al. 2004
Chain of quantum dots: conduction mechanism

When an electron leaves a dot and reaches another one:

- It has to borrow to the phonons the energy difference of states → $P \sim e^{-\Delta E/T}$;
- It has to cross impurities by tunneling. Computation for a Luttinger liquid → transparency $e^{-s}$ for one impurity, thus $(e^{-s})^n$ for cotunneling through $n$ impurities. Larkin & Lee 1978
- Decoherence after each hop (dissipation).

Competition between tunneling and activation
Equivalent resistor circuit in the low field regime

A. Miller & E. Abrahams 1960

If the applied voltage is small, current $I$ is prop. to voltage $V$ and one may define resistances :

$$R_{ij} := \frac{\xi_i - \xi_j}{I_{i \rightarrow j}} \approx R_0 e^{\frac{E_{ij}}{T} + s|j-i|} .$$

→ Resistor network (with $T$ dependance) with disorder

We performed simulations to compute the total $R$ (not completely straightforward but works quick and fine up to more than 10000 impurities).
“Percolating path” approximation

At low temperatures, it is often claimed that $R$ is given by $R$ of the “percolating path”.


▶ Our numerics $\rightarrow$ this approximation is very good up to $T \approx 100\Delta$. $\Delta$ : typical charging energy of one dot.
▶ Therefore we will discuss properties of the percolating path because it’s easier.
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Numerical insight for one sample: potential gradient

\( K = 0.1 \)
\( N = 100 \) impurities

2 regimes:
extensive / non extensive \( R \)
Experiment: Carbon NanoTubes

Mott’s Variable Range Hopping (1)

Model for disordered semiconductors:

Localized states (LS) uniformly random in space ($x$) and energy ($E$)
Hopping conductivity
Mott’s Variable Range Hopping (2) : 1D breaks

In 1D, resistance of the sample is not fixed by the typical resistance but by rare events.

Kurkijärvi PRB 1973; Raikh & Ruzin JETP 1989

Break of resistance $R_0 e^u$ has probability $\propto e^{-Tu^2}$ to occur in each position.

Strongest break that occurs in the wire of length $L$:

$$Le^{-Tu^2} \approx 1 \rightarrow u \approx \sqrt{\ln(L)/T}$$

thus total resistance is $\approx R_0 e^{\sqrt{\ln(L)/\sqrt{T}}}$. 
Breaks in our model (1)

Difference with Raikh & Ruzin : correlation between $x$ and $E$ [and strong interactions between electrons].

Probability of a break of resistance $R_0e^u$ :

$$\ln P(u) \approx \begin{cases} -\frac{1}{s} \frac{T}{\Delta} u^2 & \text{when } u \ll \frac{\Delta}{T} \\ -\frac{1}{s} u \ln \frac{uT}{\Delta} & \text{when } u \gg \frac{\Delta}{T}. \end{cases}$$

Strongest break that may be observed in a wire of length $L$ given by $u_{\text{max}}$ such that :

$$L \int_{u_{\text{max}}}^{+\infty} P(u) du \approx 1.$$
Breaks in our model (2)

Hence

\[
\text{average } \ln R \sim \begin{cases} 
\sqrt{\frac{\Delta}{T}} & \text{when } \frac{T}{\Delta} \ll \frac{1}{s} \frac{1}{\ln L} \\
\frac{s}{\ln L} \frac{\ln L}{\ln \frac{T}{\Delta}} & \text{when } \frac{1}{s} \frac{1}{\ln L} \ll \frac{T}{\Delta} \ll \frac{1}{s}
\end{cases}
\]

Breaks in our model (3)

Numerically confirmed, *but*

- The two regimes may be hard to distinguish
- Huge sample to sample fluctuations
Distribution of $\ln R$ (1)

$$R_{\text{wire}} \approx R_{\text{perc.path}} = R_1 + R_2 + \ldots + R_k$$

$$R_i \approx e^{n_i s + \frac{A_i}{T}}$$ is random \Rightarrow$$

Gaussian $R_{\text{wire}}$ if $T$ high or $L$ (thus $k$) large

$$R_{\text{wire}} \approx \max(R_i)$$ if $T$ small and $L$ not too large. Or : $\ln R_{\text{wire}} \approx \max(\ln R_i)$.

Extreme value statistics :

Fisher-Tippett-Fréchet-Weibull-Gumbel 1920’s ; Gnedenko 1941

$max(\ln R_1, \ln R_2, \ldots, \ln R_k)$ is not Gaussian ; follows (in our case) Gumbel law
Distribution of $\ln R$ (2)

Finite-size effects:

P. Hall 1978; . . . ; Z. Rácz & coworkers 2000’s

- **at low** $T$, $\ln P(u) \propto -\frac{1}{s} T u^2$ (well verified in the simulation) ⇒

  $\ln R \sim \sqrt{\ln L} \pm \frac{1}{\sqrt{\ln L}}$

  known in standard VRH: Lee et al. 1984, 1986

  $P(\ln R) = \text{Gumbel} + \text{f.-s. corrections in } 1/\ln L$

  The corrections disagree with results from Raikh & Ruzin (1989)

- **at medium** $T$, and if interactions ($K$) are not too large,

  $\ln P(u) \propto -\frac{1}{s} u \ln \frac{uT}{\Delta} \Rightarrow \ln R \sim \ln L / \ln \ln L \pm \ln \ln L / \ln L$

  $P(\ln R) = \text{Gumbel} + \text{f.-s. corrections in } 1/\ln L / \ln(s \frac{T}{\Delta} \ln L)$

  (new).

Suggestion to experimentalists: finite-size corrections to distributions can be a proof of (Luttinger liquid + strong impurities)
Distribution of $\ln R$ : numerics low $T$

K=0.1, N=1000, T/Delta=0.001

numerical data, exact $R$
Gaussian law
Gumbel law
Distribution of $\ln R$ : numerics low $T$ — f.-s. corrections

Finite-size corrections -- $K=0.1$ $N=100$ $T/Delta=0.001$

numerical exact average $\ln(R)$
Correction to Gumbel law

finite-size corrections after Raikh & Ruzin's result
Distribution of $\ln R$ : numerics medium $T$
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- Rich behaviour of (quasi) 1D quantum wires with disorder: Several temperature regimes, non trivial statistics
- Looking at the distribution of $R$ may confirm/infirm the theory $\rightarrow$ call for systematic measurements

Perspectives

- Going to the non ohmic regime (strong electric field, non linear)
- Several coupled chains
- Could these results extend to granular metal arrays?
- Studying the distribution of current and shot noise
Conclusion

- Rich behaviour of (quasi) 1D quantum wires with disorder: Several temperature regimes, non trivial statistics
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Perspectives

- Going to the non ohmic regime (strong electric field, non linear)
- Several coupled chains
- Could these results extend to granular metal arrays?
- Studying the distribution of current and shot noise

Thanks to T. Nattermann and M. Fogler!
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Effect of a single impurity / many weak impurities

\[ I \propto \begin{cases} V^{b+1} & V \gg 1 \\ T^a V & V \ll 1 \end{cases} \text{ with } a = b = 2 \left( \frac{1}{K} - 1 \right). \]

Kane & Fisher 1992

Same behaviour for many weak impurities.

Experimentally verified on short wires \((L < 1\mu m)\) or on relatively clean wires. But on some long wires \((L > 10\mu m)\), \(a/b\) is between 2 and 5: effect of strong disorder?
Model of the disorder

Strong, point-like impurities distributed randomly according to a Poisson process with cut-off. Between : pure Luttinger liquids. Probability of the distance \( l_i = x_{i+1} - x_i \) between impurities \( i \) and \( i+1 \) :

\[
P(l_i) = \frac{1}{\langle l_i \rangle} \exp\left(-\frac{l_i}{\langle l_i \rangle}\right)
\]

\[
\langle l_i \rangle = l \gg \lambda
\]

\[
\forall i \quad l_i > \lambda
\]
Hopping current between two dots (1)

A. Miller & E. Abrahams 1960

Net flow from state $i$ to state $j$:

$$I_{i\rightarrow j} = f_i(1 - f_j)w_{ij} - f_j(1 - f_i)w_{ji}$$

where $f_i$ is the occupation probability of state $i$ by an electron (fermion):

$$f_i = \left[ \exp \frac{E_i - \mu_i}{k_B T} + 1 \right]^{-1}$$

and $w_{ij}$ is the “attempt frequency” to go from $i$ to $j$:

$$w_{ij} = \text{sign}(E_j - E_i)|\gamma|e^{-s|i-j|}$$

**tunnel effect**

$$\left[ e^{\frac{E_j - E_i}{k_B T}} - 1 \right]^{-1}$$

**borrowing from phonons**
Hopping current between two dots (2)

At low temperatures,

\[ I_{i \rightarrow j} \sim I_0 e^{-s|i-j|} e^{-\frac{E_{ij}}{2T}} \sinh \frac{\xi_i - \xi_j}{2T} \]

with

\[ E_{ij} := \min_{\sigma, \tau = \pm} \left( |E_{i}^{\sigma} - \mu_i| + |E_{j}^{\tau} - \mu_j| + |E_{j}^{\tau} - E_{i}^{\sigma}| \right) \]

and, in the Ohmic (linear) regime where \( I \) is proportional to the applied field \( F \) or to the difference of electrochemical potential, we define

\[ R_{ij} := \frac{\frac{\xi_i - \xi_j}{I_{i \rightarrow j}} \approx R_0 e^{\frac{E_{ij}}{T} + s|i-j|}}{I_{i \rightarrow j}} . \]

→ We now have a disordered network of resistances which depend on \( T \).
Mott’s Variable Range Hopping (1)

Model for disordered semiconductors:

Localized states (LS) uniformly random in
space ($x$) and energy ($E$)
Hopping conductivity
Localization length $a$

Optimize typical conductivity $G = G_0 e^{-\frac{|x_j-x_i|}{a}} - \frac{\Delta E}{T}$ (with
$\Delta E = E_j - E_i > 0$):
in the ball of radius $r$ with density of states $\rho$, typically $\rho r^D$ states
$\rightarrow$ lowest $\Delta E \sim 1/(\rho r^D)$ in dimension $D$.

Optimizing over $r \rightarrow G \sim e^{-\left(\frac{T_0}{T}\right)^{\frac{1}{1+D}}}$.

In 3D, well known experimental result $R \sim e^{\left(\frac{T_0}{T}\right)^{\frac{1}{4}}}$. 
Breaks in our model (2)

Hence

\[
\ln R \approx \begin{cases} 
\sqrt{s \frac{\Delta}{T} \ln \frac{L}{l_0}} & \text{when } T \ll \frac{\Delta}{s} \frac{1}{l_0} \\
\frac{\ln \frac{L}{l_1}}{s \ln s Y \frac{T}{\Delta}} & \text{when } \frac{\Delta}{s} \frac{1}{l_0} \ll T \ll \frac{\Delta}{s} \frac{L}{\lambda}
\end{cases}
\]

\(X, Y, l_0, l_1\) : constants or slowly varying functions of \(T\).
“Phase diagram” : (non) extensivity of $R$